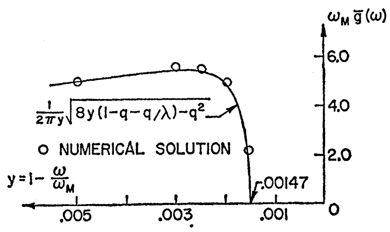

FIG. 4. Spectral function near the band edge for the case $M/M'=3$, $q=\frac{1}{10}$.

with clusters of impurities, we cannot hope to duplicate the detailed structure which we expect to exist at the high-frequency end of the spectrum.[3] In other words, the present calculation probably gives a $\bar{g}(x)$ which is suitable for calculation of the specific heat but which is not adequate for, say, transport calculations where the detailed dynamics of the system may be more important.

Apropos of the broad impurity band, we should mention that Flinn, Maradudin, and Weiss[4] have found a spectrum in remarkably good agreement with Fig. 3 using a completely different method. Also, it appears to be characteristic of the self-consistent field approximation to broaden the spectrum of allowed eigenvalues from that obtained using (4) or its analog. Klauder[5] has found this to be the case in his study of electron spectra in disordered metals.

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Field Emission in a Magnetic Field*

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An expression for the field-emission current in a longitudinal magnetic field is derived in the zero-temperature limit. Two cases are considered, corresponding to constant Fermi energy (A) and constant electron density (B). In both cases the calculated current density contains an oscillatory contribution periodic in $1/H$, as well as a term which decreases as the square of the magnetic field. In case B, however, an oscillatory contribution appears that is absent in case A. Since the two oscillatory terms in case B differ in phase and their amplitudes depend on different powers of H, it should be possible to distinguish between cases A and B. The current-decrease quadratic in H has its origin in the steady diamagnetism of the electron gas. Using accepted values of effective mass, Fermi energy, and work function, we find that for bismuth the predicted variations of the emission current with magnetic field should be readily observable.

INTRODUCTION

THE effects of a strong magnetic field upon the physical properties of metals, semimetals, and semiconductors have received considerable attention in recent years.[1] Much of the impetus derived from the lucid exposition of Lifshitz and co-workers[2] who demonstrated the far-reaching inferences that could be drawn from measurements of magnetoresistance and Hall effect on pure single crystals at low temperatures. At the same time, Harrison's work[3] provided a simple link between de Haas–van Alphen data and what had appeared to be very complicated band structures of most polyvalent metals. Finally, improved techniques of crystal purification and growth, the attainment of magnetic fields of better than 10^5 G by pulse techniques, and the development of improved experimental techniques account for the rapid accretion in recent years of de Haas–van Alphen, Shubnikov–de Haas, cyclotron resonance, and related data on a host of conductors.[4]

Application of a magnetic field to a free-electron gas gives rise to highly degenerate energy levels separated by $\hbar\omega=\beta^*H=e\hbar H/m^*c$ as well as to regular singularities in the density-of-states function, thereby exerting a profound influence on any physical property either directly or indirectly related to the electronic system. Variations of the magnetic susceptibility, of the specific heat, and of the transport properties periodic in H^{-1} are the direct effects most frequently investigated. The only indirect effect that has been studied is the influence of a magnetic field on the velocity of sound.[5]

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[2] I. M. Lifshitz and V. G. Peschanskii, Zh. Eksperim. i Teor. Fiz. **35**, 1251 (1958); **38**, 180 (1960) [translations: Soviet Phys.—JETP **8**, 875 (1959); **11**, 131 (1960)]. N. E. Alekseevskii, Yu. P. Gaidukov, I. M. Lifshitz, and V. G. Peschanskii, *ibid.* **39**, 1201 (1960) [translation: *ibid.* **12**, 837 (1961)].

[3] W. A. Harrison, Phys. Rev. **126**, 497 (1962); **118**, 1190 (1960); **116**, 555 (1959).

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On the following pages we focus attention on yet another direct effect which, as far as we know, has not been the subject of either theoretical or experimental investigation, namely, the current emitted from a cold metallic surface in a strong electric field. The periodic variations in χ, C_v, and the transport properties with magnetic field arise because $N(\eta)$, the density of states at the Fermi energy η, exhibits singularities at intervals periodic in H^{-1}, and all of the aforementioned properties depend critically upon $N(\eta)$. By contrast, in high-field emission *all* the conduction electrons can contribute to the current although the probability of emission is greater for electrons of higher energy. The observed current is thus a suitable integral over the electron distribution, and, consequently, we would expect the oscillations in H^{-1} to be somewhat less well defined. Nevertheless, as we shall see, periodic variations of the emission current with magnetic field should be readily observable under appropriate, physically attainable conditions.

The phenomenon we consider here bears some similarity to current oscillations in tunnel diodes in strong longitudinal magnetic fields.[6] In that case, the current oscillations arise, indirectly, from oscillations of the electron Fermi level which brings forth corresponding changes in the junction field.[7] Since the barrier width in a tunnel diode is roughly independent of energy in the energy range of interest, the electrons that make the dominant contribution to the tunnel current are those in the lowest orbital quantum states. In our case, we face a somewhat different situation. The width of the barrier increases with decreasing electron energy and normally only electrons near the Fermi energy contribute to field emission.[8]

CALCULATION OF THE EMISSION CURRENT

The allowed energy levels of an electron in a magnetic field, chosen along the z direction, are given by[9]

$$\epsilon=\epsilon_l(k_z)=\epsilon_l+\epsilon_z=\hbar\omega(l+\tfrac{1}{2})+\hbar^2k_z^2/2m^*, \tag{1}$$

where

$$\omega=eH/m^*c \tag{2}$$

is the cyclotron frequency of electrons of effective mass m^* and l is a positive integer or zero.

The number of states with quantum number l and energy between ϵ and $\epsilon+d\epsilon$ is

$$N_l(\epsilon)d\epsilon=\frac{2eH}{h^2c}(2m^*)^{1/2}(\epsilon-\epsilon_l)^{-1/2}d\epsilon. \tag{3}$$

[6] A. G. Chynoweth, R. A. Logan, and P. A. Wolff, Phys. Rev. Letters **5**, 548 (1960).

[7] R. R. Haering and P. B. Miller, Phys. Rev. Letters **6**, 269 (1961).

[8] R. H. Good and E. W. Mueller, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), 2nd ed., pp. 176, 231.

[9] We shall disregard spin splitting throughout this discussion.

The emitted current density is given by the product of the flux of electrons of energy ϵ incident on the surface of the metal from within and the penetration probability D integrated over the entire electron distribution. The flux of electrons with energy about ϵ, $v_z>0$, and quantum number l is

$$\tfrac{1}{2}f(\epsilon)v_{z,l}(\epsilon)N_l(\epsilon)d\epsilon,$$

where $f(\epsilon)$ is the Fermi distribution and the factor $\frac{1}{2}$ takes account of the fact that for given ϵ only half the electrons have a positive z component of velocity. We, thus, are led to the following expression for J_l, the current density attributable to the lth orbital level:

$$J_l=\int_{\epsilon_l}^{\infty}\frac{2e^2H}{h^2c}f(\epsilon)D_l(\epsilon)d\epsilon. \tag{4}$$

Finally, the total emission current density is obtained by summing over all orbital states; i.e., $J=\sum_l J_l$.

We now proceed on the assumption that the penetration probability in a longitudinal magnetic field is the same as in zero field. Accordingly, $D_l(\epsilon)=D_l(\epsilon_z,F)$ is given to good approximation by[8]

$$\begin{aligned}D_l(\epsilon_z,F)&=\exp\left[-g-\frac{\eta-\epsilon_z}{d}\right]\\&=\exp\left[-g-\frac{\eta-\epsilon+\hbar\omega(l+1/2)}{d}\right],\end{aligned} \tag{5}$$

where

$$g=\frac{6.83\times10^7\phi^{3/2}}{F}v\left(3.79\times10^{-4}\frac{F^{1/2}}{\phi}\right), \tag{6}$$

$$d=\frac{9.76\times10^{-9}F}{(m^*/m)^{1/2}\phi^{1/2}t(3.79\times10^{-4}F^{1/2}/\phi)}. \tag{7}$$

Here, F is the electric field in V/cm at the surface of the metal; ϕ is the work function in eV; $v(y)$ and $t(y)$ are functions evaluated by Burgess, Kroemer, and Houston[10] and shown in Fig. 1.

From (4) and (5) we obtain the simple result

$$J=A\sum_l B_l\int_{\epsilon_l}^{\infty}f(\epsilon)e^{\epsilon/d}d\epsilon, \tag{8}$$

where

$$A=\frac{2e^2H}{h^2c}e^{-[g+\eta/d]}, \tag{8a}$$

$$B_l=e^{-\epsilon_l/d}. \tag{8b}$$

The integral in Eq. (8) is of a type commonly encountered and is conveniently evaluated in a series in powers of kT/η. In the present treatment we restrict ourselves to the limit $T\rightarrow0$ and retain only those non-

[10] R. E. Burgess, H. Kroemer, and J. M. Houston, Phys. Rev. **90**, 515 (1953).

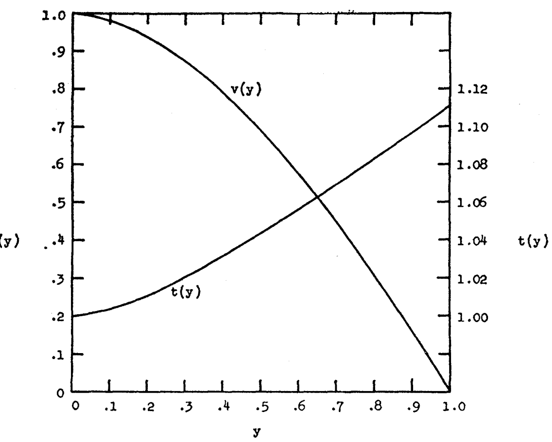

FIG. 1. The functions $v(y)$ and $t(y)$.

vanishing terms of lowest order in the expansion parameter. In this approximation,

$$\begin{aligned} J &= A \sum{}' B_l d[e^{\eta/d} - e^{\epsilon_l/d}] \\ &= A d \sum{}' [e^{(\eta-\epsilon_l)/d} - 1]. \end{aligned} \tag{9}$$

The prime on the summation denotes that the sum is to be taken over all values of l between 0 and $l_{\max}$, where $l_{\max}$ is given by $l_{\max} = \eta/\hbar\omega - \frac{1}{2}$. With the aid of the Poisson summation formula, one readily obtains

$$J = Ad\left[\frac{d(e^{\eta/d}-1)-\eta}{\hbar\omega} + \sum (-1)^p \left\{\left[\frac{2}{4\pi^2p^2+(\hbar\omega/d)^2}\right] \times\left[2\pi p \sin(2\pi p\eta/\hbar\omega) - \left(\frac{\hbar\omega}{d}\right)(\cos(2\pi p\eta/\hbar\omega) - e^{\eta/d})\right] - \frac{1}{\pi p}\sin(2\pi p\eta/\hbar\omega)\right\}\right]. \tag{10}$$

The oscillations in J with H, periodic in $1/H$, are apparent from Eq. (10).

We here distinguish between two situations which often may not be realized in practice but which represent extreme limits.

(A) The light-mass conduction band overlaps a heavy-mass hole band, and (B) there is no band overlap whatever.

Case A is approximated by many semimetals, such as bismuth, in which de Haas–van Alphen oscillations are most easily observed. Case B is probably rare in all but monovalent metals, but may be approximated in suitably doped n-type semiconductors, for example, n-InSb.

In case A, the high density-of-states hole band will maintain a fixed Fermi level by accommodating electrons from or contributing them to the electron band as the magnetic field is varied. In case B, the number of electrons will remain fixed and the Fermi energy now depends on the strength of the applied magnetic field.

In case A, Eq. (10) represents the final result. If, however, n is fixed, additional oscillatory terms appear that arise from the variation of η with H. At constant electron concentration, the Fermi energy in a magnetic field is given by[11]

$$\eta = \eta_0 \left\{ 1 - \frac{1}{12}\left(\frac{\pi kT}{\eta_0}\right)^2 - \frac{1}{80}\left(\frac{\pi kT}{\eta_0}\right)^4 + \frac{1}{48}\left(\frac{\hbar\omega}{\eta_0}\right)^2 + \frac{1}{384}\left(\frac{\pi kT\hbar\omega}{\eta_0^2}\right)^2 - \frac{\pi kT(\hbar\omega)^{1/2}}{\sqrt{2}(\eta_0)^{3/2}} \sum_{q=1}^{\infty} \frac{(-1)^q}{\sqrt{q}} \frac{\sin\lambda}{\sin\xi} \right\}; \quad kT < \eta_0,\ \hbar\omega < \eta_0, \tag{11}$$

where η_0 is the Fermi energy at $T = 0$°K, $H = 0$, and λ and ξ are given by

$$\begin{aligned} \lambda &= 2\pi q\eta/\hbar\omega - \pi/4, \\ \xi &= 2\pi^2 qkT/\hbar\omega. \end{aligned} \tag{12}$$

In the zero-temperature limit, to which we are restricting our treatment, Eq. (11) reduces to

$$\eta(H) = \eta_0 \left\{ 1 + \frac{1}{48}\left(\frac{\hbar\omega}{\eta_0}\right)^2 - \frac{1}{(8\pi^2)^{1/2}}\left(\frac{\hbar\omega}{\eta_0}\right)^{3/2} \times \sum_{q=1}^{\infty} \frac{(-1)^q}{\sqrt{q}} \sin\lambda \right\}. \tag{13}$$

It is the Fermi energy $\eta(H)$, as given by Eq. (13), which must be substituted in Eq. (10) when evaluating the emission current for case B. Although the field dependence of the Fermi energy through the monotonic increase with H^2 and the oscillatory terms is relatively small, nevertheless, this effect cannot be neglected, particularly in the first term of Eq. (10) where the Fermi energy appears in the exponent. Provided $\hbar\omega/\eta_0 \ll 1$, it is permissible to replace η by η_0 in the expression for λ, Eq. (12), and also in the arguments of the trigonometric function in Eq. (10). This simplifying approximation cannot be employed at fields of sufficient strength such that $\hbar\omega \gtrsim \eta_0$, the prevailing situation already at moderate fields ($\sim$15 kG) in bismuth and at even lower fields in dilute BiSb alloys.

NUMERICAL EVALUATION OF THE EMISSION CURRENT

In this section we present results of a numerical evaluation of Eq. (10) for bismuth using reasonable values of electric field and other parameters. The preferred value of the work function of bismuth is 4.25 eV[12]; for the Fermi energy η_0 and effective mass m^* we take 15.7×10^{-3} eV and 0.0105 m_0[13]; for the electric field we select a representative value of 3×10^7 V/cm. One

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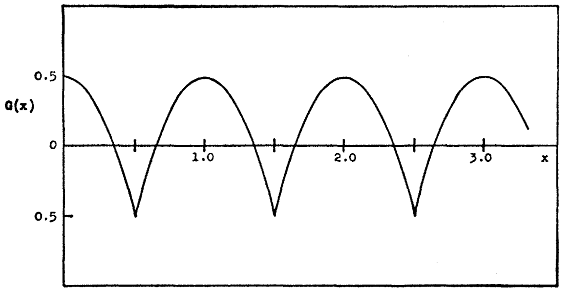

FIG. 2. The function $G(x)$.

obtains

$$g=1.43, \quad d=1.33 \text{ eV}$$

and

$$\beta^*=1.1\times10^{-3} \text{ eV/kG}.$$

From these numerical values it follows directly that for reasonable magnetic fields, less than 50 kG, say,

$$\hbar\omega/d\ll1 \text{ and also } \eta_0/d\ll1.$$

We may, therefore, use these ratios as expansion parameters in the evaluation of Eq. (10). To lowest order in $\hbar\omega/d$, η_0/d, we obtain

$$J=A'\left\{1-\frac{1}{8}\left(\frac{\hbar\omega}{\eta_0}\right)^2+\frac{1}{4}\left(\frac{\hbar\omega}{\eta_0}\right)^2G\left(\frac{\eta_0}{\hbar\omega}\right)\right\}, \qquad (14)$$

where

$$A'=\frac{2\pi em^*\eta_0^2}{h^3}e^{-g}.$$

$G(x)$ is the periodic function

$$G(x)=\tfrac{1}{2}-4x^2, \quad -\tfrac{1}{2}<x<\tfrac{1}{2}$$

shown in Fig. 2.

Since bismuth approximates case A (constant n), Eq. (14) is the desired result. At fields near 10 kG oscillations whose amplitudes are about 10% of the average current should appear.

For comparison, we give also the result for case B (constant n) using the same numerical parameters. We obtain then, retaining only terms to order $(\hbar\omega/\eta_0)^2$,

$$J=A'\left\{1-\frac{1}{12}\left(\frac{\hbar\omega}{\eta_0}\right)^2+\frac{1}{4}\left(\frac{\hbar\omega}{\eta_0}\right)^2G\left(\frac{\eta_0}{\hbar\omega}\right)+\left(\frac{1}{2\pi^2}\right)^{1/2}\left(\frac{\hbar\omega}{\eta_0}\right)^{3/2}F\left(\frac{\eta_0}{\hbar\omega}\right)\right\} \text{ A/cm}^2, \qquad (15)$$

where

$$F\left(\frac{\eta}{\hbar\omega}\right)=\sum_{p=1}^{\infty}(-1)^p\frac{\sin(2\pi p\eta/\hbar\omega-\pi/4)}{\sqrt{p}}.$$

For the parameters we have selected $A'\simeq5000$ A/cm^2.

Of the three field-dependent terms in (15), the last clearly dominates at low fields; i.e., when $\hbar\omega\ll\eta_0$. As the magnetic field is increased and $\hbar\omega$ approaches η_0, the term involving the function $G(\eta_0/\hbar\omega)$ takes on increasing importance. Since $G(\eta/\hbar\omega)$ and $F(\eta/\hbar\omega)$ differ in phase by $\pi/4$, and the amplitudes of the oscillation have a slightly different field dependence, it should be possible to identify the two experimentally. Finally, we anticipate a monotonic decrease in J with H, given by the term $\frac{1}{12}(\hbar\omega/\eta_0)^2$, which has the same origin as the steady diamagnetic susceptibility of the electron gas.

CONCLUSION

We have investigated theoretically the variation of the high-field emission current in a longitudinal magnetic field in the zero-temperature limit within the single-particle free-electron approximation. Under suitable conditions, perhaps most easily realized in bismuth and bismuth-antimony alloys, the emission current should show oscillations of the de Haas–van Alphen type. Moreover, we anticipate a monotonic decrease in emission, quadratic in H.

The expressions for the emission current in the cases of constant Fermi energy and constant electron concentration differ through the presence, in the latter instance, of an additional oscillatory term whose phase and field dependence set it apart from the term which alone determines the oscillatory behavior in the former case. Since a fixed Fermi energy implies the presence of an overlapping high density-of-states band, emission current variations, apart from their intrinsic interest, may provide useful information on the band structure. It may also develop that the dependence of the emission current on magnetic field could prove valuable in the study of surface effects in semimetals and some semiconductors.

In our derivation of the equation for the emission current, we have assumed that the penetration probability, D, does not depend explicitly on the magnetic field. This assumption cannot be justified either theoretically or by recourse to experimental data since such is, as yet, nonexistent. It may well be that the function $D(\epsilon_z,F)$ depends on H also; this would surely modify the behavior profoundly, but at present it seems futile to try to anticipate that contingency.

Note added in proof. The author would like to thank Dr. N. Goldberg and Dr. I. Pollak for calling to his attention an error in sign in Eq. (10) of the original manuscript.